**Multivariable Calculus - Test 1 Fall 2015**

**Answers**

**Part 1**

In , let , and .

1. Give the parametric form of the line through and .

We see . So the parametric form of the line is:

1. Give the equation of the plane containing the line in part A and the point

This plane contains the vector and . So, we take as a normal vector to this plane So the equation of the plane is:

1. Give the vector form of a line through that lies in the plane described in part B and is orthogonal to the line described in part A.

The vector for this line must be orthogonal to since the lines are perpendicular; it must be perpendicular to since it is on the plane. So, we take to be

. So the line is described by:

**Part 2**

Let be the space curve defined by .

1. At what points does meet the -plane?

The curve meets the plane when the -coordinate of is zero. Setting yields . Plugging these into gives us the points: and .

1. Give the vector form of the line tangent to at .

Since, , we know the point is on the line. To find the vector along the line we use . Since , we know the line is given by:

1. Does the line in Part B intersect the line defined by ? Explain.

To see if these lines meet, we need to see if there are numbers and so that . We try to solve the system:

This system has the solution: . Therefore, the point: is on both lines.

**Part 3**

1. Let be a function. Give the definition of the partial derivative, .
2. Give the equation of the plane tangent to the graph of at (3,0).

Since , we see that the point is on the plane. We need to evaluate the partial derivatives of at . To this end, we see:

So,

So, the equation of the plane tangent to the graph of is: .

1. Give two non-parallel vectors that line on the plane described in Part B. Briefly explain.

A vector will be on the plane if it is orthogonal to the normal vector for the plane which we see is: . There are many answers to this question. The two we used in class to derive the equation of the tangent plane are: and .

**Part 4**

Let .

1. Sketch the level set of containing the point (3,12,1). What is the corresponding level?

Since , the level is . The level set has the equation: . This is equivalent to: which describes an elliptical paraboloid.



1. Give the equation of the plane tangent to the graph of this level set at the point (3,12,1).

We take as the normal vector for this plane the gradient of evaluated at . To that end, we see, , so . So, the equation of the plane is:

1. Give the parametric form of the line through the origin which is normal to the plane found in part B.

Here, we are asked for the line through the origin along the vector which is:

**Part 5**

A two dimensional coordinate system is placed over a lake. The depth in meters of a point in a lake is given by the function. A duck is floating at the position .

1. As she begins to swim towards a frog at ), does the water get more deep or less deep for the duck? Explain.

This question asks for a rate of change in a direction, so the answer involves calculating a directional derivative. The vector from the duck to the frog is: . The unit vector in this direction is: . The gradient of is given by . Evaluating this at the duck’s point yields: . So, the directional derivative is given by . Since the rate of change is negative, it means the depth is decreasing. That is to say, the water gets shallower.

1. A pig is swimming in the lake at point . In which direction should he swim so that the water gets as shallow as it immediately can?

The gradient points in the direction of maximal increase; the vector opposite the gradient points in the direction of maximal decrease. Here, that is: .

1. A macaque is swimming so that the coordinate of his position is always increasing and so that the depth of the water under him is always 11 meters. In which direction is he swimming as he passes the point ?

There are a few ways of answer this question. The most straightforward might be to realize that the vectors orthogonal to the gradient point in directions in which the output of the function doesn’t change. In our case, the gradient for the position of the macaque is . The two directions orthogonal to this are: and . Since the coordinate of the macaque is increasing, as he passes he must be swimming in the direction of .